Math 45 4.6 Systems of Linear Inequalities

Objectives: 1) Determine if an order pair (point) is a solution of a system
2) Graph a system of linear inequalities
3) Word problems with linear inequalities.

① Determine if $(2,1)$ is a solution of $\begin{cases} 2x+y \leq 7 \\ 7x-2y \geq 4 \end{cases}$.

Step 1: Substitute given point into first inequality.
If true result, go on to step 2.
If false result, answer is no.

$2(2)+(1) \leq 7$
$4+1 \leq 7$
$5 \leq 7$ true, continue

Step 2: Substitute given point into second inequality
If true result, yes.
If false result, no.

$7(2)-2(1) \geq 4$
$14-2 \geq 4$
$12 \geq 4$ true

YES, (2,1) is a solution of this system

② Determine if $(-2,5)$ is a solution of $\begin{cases} 2x+y \leq 7 \\ 7x-2y \geq 4 \end{cases}$

Step 1: $2(-2)+5 \leq 7$
$-4+5 \leq 7$
$1 \leq 7$ true, continue.

Step 2: $7(-2)-2(5) \geq 4$
$-14-10 \geq 4$
$-24 \geq 4$ false

No, (-2,5) is not a solution of this system

Just as with systems of linear equations, the solution to a system of linear inequalities must be a solution of all inequalities in the system.

③ Graph $\begin{cases} y \geq 3x-5 & A \\ y \leq -2x+5 & B \end{cases}$

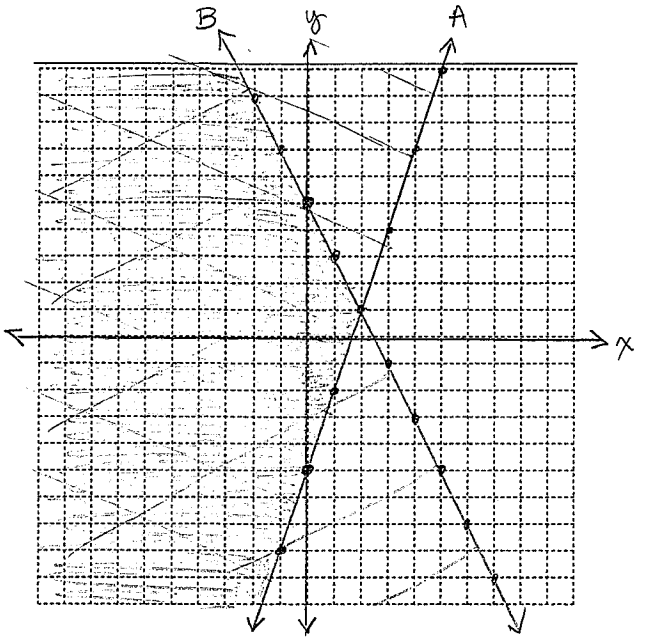
step 1: Graph A.

A: $\geq$ solid line
slope 3
y-int −5
test (0,0): $0 \geq 3(0)-5$
$0 \geq -5$ true

step 2: Graph B.

B: $\leq$ solid line
slope −2
y-int 5
test (0,0): $0 \leq -2(0)+5$
$0 \leq 5$ true

step 3: Shade the overlap.



④ Graph $\begin{cases} 2x+y < 7 & A \\ 7x-2y > 4 & B \end{cases}$

step 1: Graph A

$2x+y < 7$
$y < -2x+7$

$\begin{cases} < \text{ dotted} \\ m = -2 \\ y\text{-int } 7 \end{cases}$

test (0,0):
$2(0)+0 < 7$
$0 < 7$ true

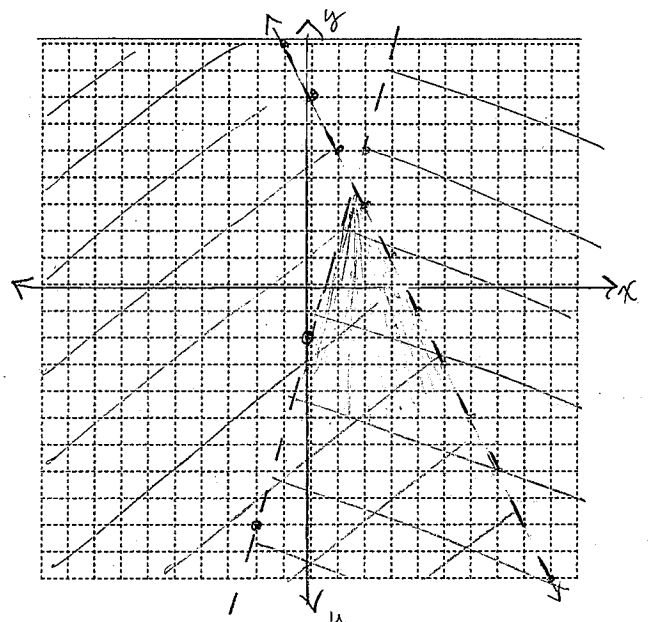
step 2: Graph B.

$7x-2y > 4$
$\frac{-2y}{-2} > \frac{-7x}{-2} + \frac{4}{-2}$
$y < \frac{7}{2}x - 2$

$\begin{cases} > \text{ dotted} \\ m = \frac{7}{2} \\ y\text{-int } -2 \end{cases}$

test (0,0)
$7(0)-2(0) > 4$
$0 > 4$ false.

step 3: Shade the overlap.

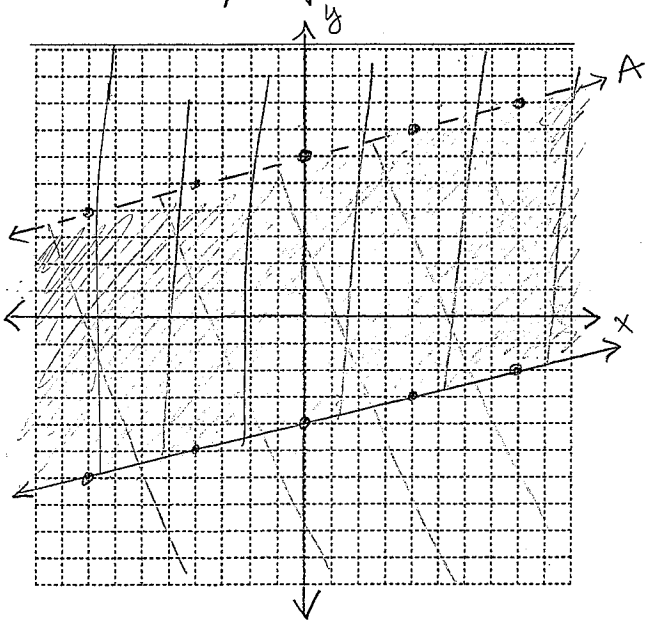


⑤ Graph $\begin{cases} y < \frac{1}{4}x+6 & A \\ y \geq \frac{1}{4}x-4 & B \end{cases}$

A: dotted, slope $\frac{1}{4}$, y-int 6
$0 < \frac{1}{4}(0)+6$
$0 < 6$ true

B: solid, slope $\frac{1}{4}$, y-int −4
$0 \geq \frac{1}{4}(0)-4$
$0 \geq -4$ true

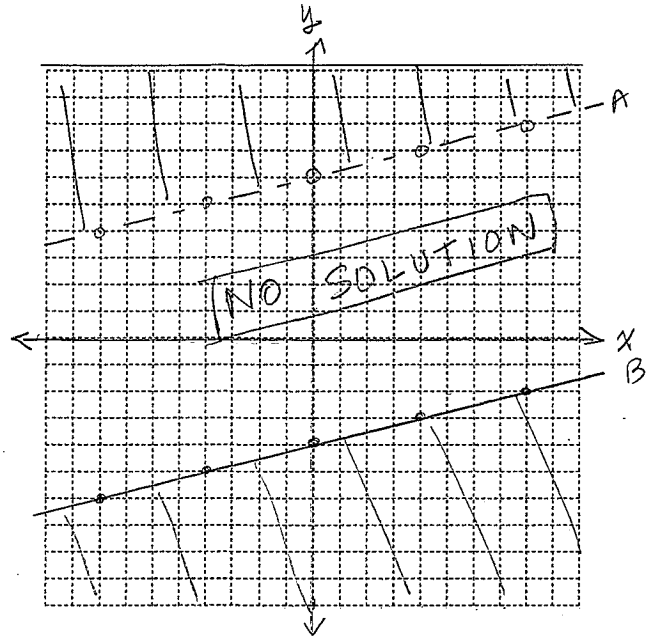
shade overlap



⑥ Graph $\begin{cases} y > \frac{1}{4}x + 6 & A \\ y \leq \frac{1}{4}x - 4 & B \end{cases}$

A: dotted, slope $\frac{1}{4}$, y-int 6.
test (0,0) $0 > \frac{1}{4}(0)+6$
$0 > 6$ false

B: solid, slope $\frac{1}{4}$, y-int -4
test (0,0) $0 \leq \frac{1}{4}(0)-4$
$0 \leq -4$ false



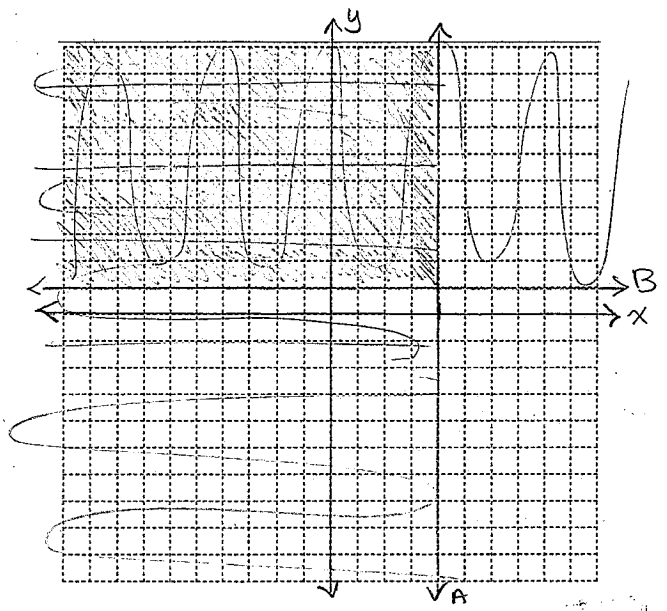
NO OVERLAP! So **no solution**.
There are no ordered pairs that make both inequalities true.

⑦ Graph $\begin{cases} x \leq 4 \\ y > 1 \end{cases}$

Step 1: $x \leq 4$ vertical, solid
test (0,0) $0 \leq 4$ true

Step 2: $y > 1$ horizontal, dotted
test (0,0) $0 > 1$ false

Step 3: shade overlap



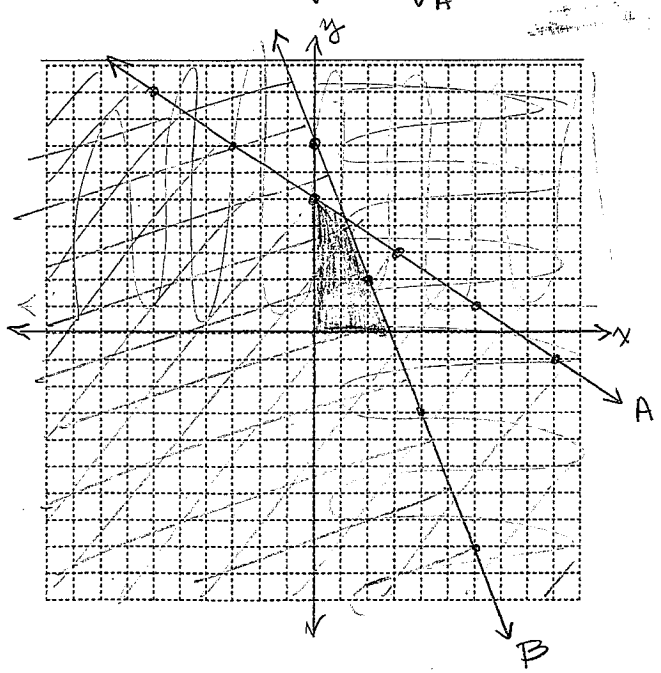
⑧ Graph $\begin{cases} y \leq -\frac{2}{3}x + 5 & A \\ y \leq -\frac{5}{2}x + 7 & B \\ x \geq 0 & C \\ y \geq 0 & D \end{cases}$

A: solid, $m = -2/3$, y-int 5
$0 \leq -2/3(0) + 5$ $0 \leq 5$ true

B: solid, $m = -5/2$, y-int 7
$0 \leq -5/2(0) + 7$ $0 \leq 7$ true

C: solid, vertical, same as y-axis.
test (1,0). $1 \geq 0$ true

D: solid: horizontal, same as x-axis
test (0,1) $1 \geq 0$ true



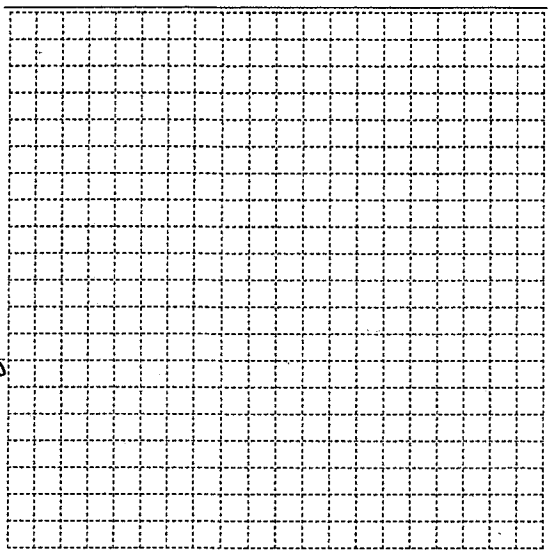
⑨ You have up to $50,000 to invest. Your advisor says to put at least $10,000 in Treasury notes (x), and no more than $35,000 in corporate bonds (y). This gives the system of linear inequalities

$$\begin{cases} x+y \le 50{,}000 & A \\ x \ge 10{,}000 & B \\ y \le 35{,}000 & C \end{cases}$$

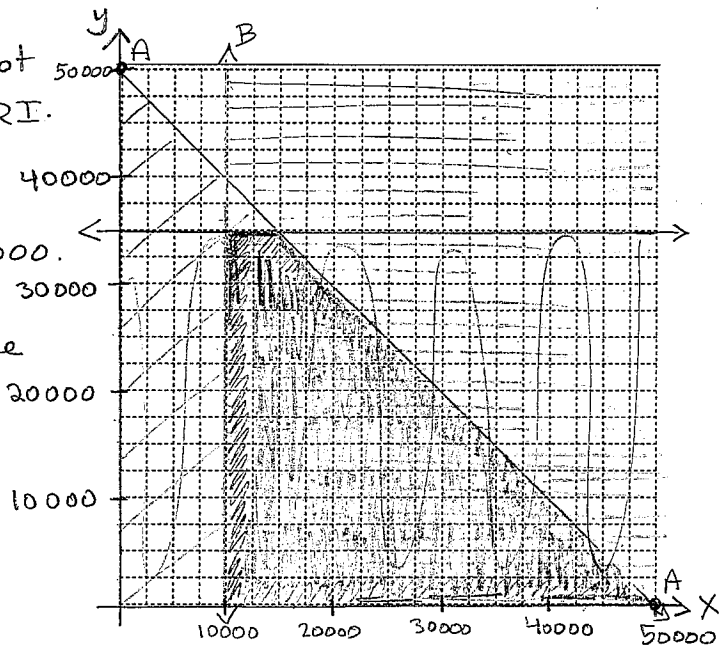
a) Graph the system of inequalities

b) Can you put $20,000 in bonds and $30,000 in treasuries?

c) Can you put $40,000 in bonds and $10,000 in treasuries?



a) Negative values of x or y do not make sense, so we draw only QI.



A: $x+y \le 50{,}000$

$y \le -x+50{,}000$

slope -1, y-int 50000.

x-int: 50,000 also.

test (0,0) $0+0 \le 50000$ true

B: $x \ge 10{,}000$

vertical, solid.

test (0,0) $0 \ge 10{,}000$ false

C: $y \le 35{,}000$

horizontal solid.

test (0,0) $0 \le 35{,}000$ true

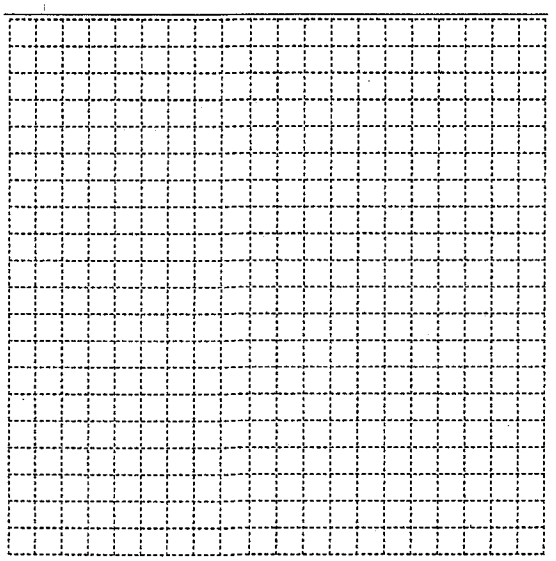
b) $\left.\begin{matrix} x = 30000 \\ y = 20000 \end{matrix}\right\}$

$20000 + 30000 \le 50000$ ✓

$30000 \ge 10000$ ✓

$20000 \le 35000$ ✓

YES



c) $\left.\begin{matrix} x = 10{,}000 \\ y = 40000 \end{matrix}\right\}$

$10000 + 40000 \le 50000$ ✓

$10000 \ge 10000$ ✓

$40000 \le 35000$ ✗

NO

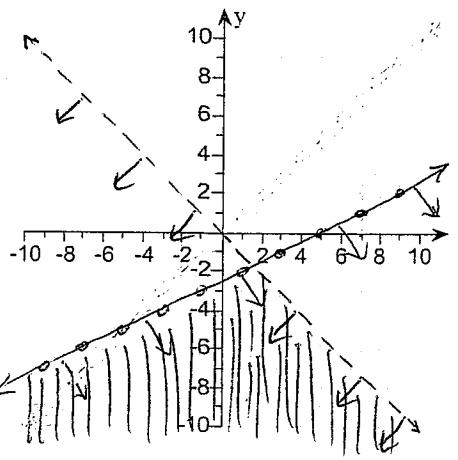
Math 45

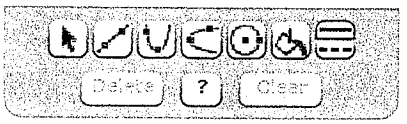
4.6.33 Graph the system of linear inequalities.

$$\begin{cases} x \geq 5y+2 \\ x+y < 0 \end{cases}$$

Use the graphing tool to graph the system.

Click to enlarge





To graph $x \geq 5y+2$

$x-2=5y$

$5y=x-2$

$y=\frac{1}{2}x-\frac{2}{5}$ ← cannot be graphed on MathXL.

Use techniques from class today:

$5y=x-2$

choose x so that $x-2$ is a multiple of 5.

$x=7$ $\quad 5y=7-2$

$5y=5$

$y=1$ $\quad (7,1)$ plot this, then use slope $\frac{1}{2}$.

Test (0,0)

$0 \geq 5(0)+2$

$0 \geq 2$

false. (0,0) is not a solution. Shade down

$x+y<0$

Graph $y=-x$ with dotted line.

(0,0) is on this line. Choose a different test point.

(1,0) is not on the line. Test (1,0)

$1+0<0$

$1<0$ false. (1,0) is not a solution. shade down.

Shade overlapping shaded areas.